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This report summarizes the research activities carried out under the subject grant during the period May 1981 to April 1982. Specifically, using a variational principle with varying boundaries, it was shown that both physical and material conservation laws can be obtained from the same expression. The basic quantities used in these conservation laws are the physical momentum (stress) and material momentum. Two different repøresentations were employed to study the equal importance of both of them in elastic materials and structures.
In addition, by a suitable choice of the Lagrangian, the balance laws of moment of material momentum was derived directly by considering rotations in material space. This permitted then also to derive the balance law connected with similarity and establish the relation between these balance laws and path-independent integrals of fracture mechanics. Further, a suitable form of the Lagrangian was found which permits to establish conservation laws for thermoelasticity.

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ANNUAL SCIENTIFIC REPORT

covering the period

May 1, 1981 to April 30, 1982

of work performed under
Grant No. AFOSR-80-0149

PATH-INDEPENDENT INTEGRALS AND FRACTURE MECHANICS

at

Stanford University

Research sponsored by

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

submitted by

Professor Alicia G. Herrmann

Professor George Herrmann

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Stanford, California 94305

April 1983

INTRODUCTION

During the period covered by this report, considerable advances have been made in furthering our knowledge of path-independent integrals and their relation to fracture mechanics. These advances are briefly reviewed in the body of this report, making reference to published papers and to manuscripts under preparation. In order to place the research efforts carried out during the reporting period into a proper perspective, it was deemed desirable to restate the general goals and aims as established in the original proposal.

THE GENERAL AIMS AND GOALS

The rational and economic design of aerospace structural systems and components presupposes the ability to perform a fracture mechanics analysis in an efficient and reliable manner. Such an analysis, to assure structural integrity, has to take account of complex loadings (both static and dynamic) as well as thermal and other environmental conditions to which many such systems are subjected.

Of central importance in failure prevention is the quantitative knowledge, assuming the existence of cracks of subcritical size is allowed, of the so-called crack-extension force (or energy release rate), whose magnitude controls the onset of crack propagation. To this end one does not require the knowledge of the complete stress field in the structural element, but rather one needs to know only the stress intensity factor which is related to the crack extension force.

During the last decade, the calculation of crack extension forces in elastic bodies and also their experimental determination, have often been performed with the aid of the so-called J-integral. In a plane problem the J-integral is a line integral which, if taken along a closed contour, will give information regarding defects (such as cracks) within the contour. If the contour contains no defects or imperfections (such as inclusions or voids), the value of J will be zero.

If the contour contains a single crack tip, the value of J will be identical to the crack extension force. In this case the J-integral is also said to be path-independent, because whatever the contour surrounding the single crack tip, its value remains the same. In three dimensions the J.integral is a three-dimensional vector defined as an integral over a closed surface.

The J-integral is one of several so-called conservation integrals which have been recently established. The various early contributions which led to the state-of-the-art were reviewed in the original proposal (dated November 1979) and will not be summarized here. It is deemed sufficient to emphasize here that at the time of the initiation of the subject grant, various points concerning the foundation upon which the path-independent integrals were developed were in need of clarification. In addition, the existing state-or-the-art at the time dealt only with isothermal, essentially linear and static elastic behavior. Thus, extensions and generalizations to dynamics, to nonlinear elasticity and to thermo-elasticity had to be considered.

As stated in the proposal, it was believed that this work would lead to a considerable increase of our analytical tools and capabilities in dealing with fracture and defect mechanics, thus insuring a higher degree of reliability in fracture prevention and fracture control of aerospace structures.

WORK COMPLETED AND IN PROGRESS UNDER THE SUBJECT GRANT

During the period covered by this report, several papers were published which represent a significant advance in the state-of-the-art briefly summarized in the preceding section.

In ref. [1], the material momentum tensor was introduced in such a way that

it can be placed on equal footing with the physical momentum tensor (which in many cases is simply the stress tensor). This was accomplished by applying a variational principle with varying boundaries and deducing (for the first time) both physical and material conservation laws from the same expression.

In a companion paper [2], the material momentum tensor was used to establish directly path-independent integrals of fracture mechanics. Specifically, by suitable choice of the Lagrangian, the balance law of moment of material momentum is derived directly by considering rotations in material space. It is to be emphasized that the results established in this paper apply not only to static, but also to dynamic as well as nonlinear behavior.

During the reporting period, work was also in progress on an alternate method to derive, in a unified fashion, balance laws of continuum mechanics, i.e., the equations of motion and conservation laws. This method is based on a purely local character of the energy momentum and leads directly to differential equations rather than integral relations.

Special attention was devoted to the problem of a plane crack in a homogeneous, static stress field. Specifically, it was shown that the component of the J_i-integral normal to the plane of the crack (J_2) is not path-independent in the sense of the well-known J-integral parallel to the plane of the crack. The relation between the energy-release rate for rotation and the J_2 integral were established. Thus, considerable advances were made in the characterization of defects such as flaws. A paper describing this work is in preparation [3].

Considerable progress during the reporting period has also been made on the material momentum and conservation laws for thermoelasticity using variational formulation. The dissipative character of thermoelastic systems precludes the generation of the required Lagrangian function by simple addition of new terms,

involving the fields (which describe the thermal behavior) to the elastic Lagrangian. As long as the interest centers in deriving the thermoelastic equations only, it is possible to overcome the obstacle of using adjoint fields as well as original ones in the Lagrangian. Such a formulation, however, is not capable of providing the new conservation laws because the crucial quantity appearing there involves the Lagrangian itself. An alternate formulation has been found for the Lagrangian which is non-local in time and involves a symmetrized convolution in time. Thus, the material momentum is becoming time-dependent. The balance of material momentum is integrated over time to produce a relation reminiscent of the impulse-momentum equation in classical mechanics. A paper describing this work is to be published in the Journal of Applied Mechanics [4].

A more general variational approach to a Lagrangian formulation of continuum mechanics, suitable for both isothermal solids and fluids has provided additional insight into characterization of defects. A paper entitled "On the Lagrangian Formulation of Continuum Mechanics," is under preparation.

PUBLICATIONS

1. A. Goleblewska Herrmann, "On Physical and Material Conservation Laws," *Proceedings of the IUTAM Symposium on Finite Elasticity*, pp. 201-209, in press.
2. Alicia Goleblewska Herrmann, "Material Momentum Tensor and Path-Independent Integrals of Fracture Mechanics," *International Journal of Solids and Structures*, Volume 18, No. 4, pp. 319-326, (1982).
3. A. G. Herrmann and G. Herrmann, "On the Path-Dependence of the J_2 Integral," in preparation, to be published.

4. G. Francfort and A. Golebiewska Herrmann, "Conservation Laws and Material Momentum in Thermoelasticity," to be published in the *Journal of Applied Mechanics*.

5. A. G. Herrmann, "On the Lagrangian Formulation of Continuum Mechanics," to be published.

PERSONNEL

In addition to the principal investigators, the following graduate students (Ph. D. Candidates) were associated with this project.

G. Francfort

E. Pak

INTERACTIONS (Coupling Activities)

1. ASME/ASCE Joint Meeting, Boulder, Colorado, June 1981
   Invited paper entitled "On the Meaning of the J_2 Integral for a Plane Crack," presented by A. G. Herrmann
2. Euromech Symposium on "Damage Mechanics," Cachan, France, Sept. 1981.
   Invited paper entitled "Review of Recent Work on Fracture at Stanford," presented by George Herrmann.
3. ARO - Sponsored Mechanics Research Center, University of Wisconsin, Madison, Wisconsin.
   George Herrmann: Invited to peer review meeting, Sept. 1981.
4. Sixth Brazilian Congress of Engineering Mecahnics, Rio de Janeiro, December 1981. Invited paper on fracture mechanics, delivered by George Herrmann.

5. Invited lecturer at the University of Cairo, Egypt, George Herrmann, Jan. 1982.

6. Lehigh University, Bethlehem, Pennsylvania, "Path-Independent Integrals for a Plane Crack and Their Relation to General Conservation Laws in Continuum Mechanics," presented by A. G. Herrmann, March 1982.

7. Harvard University, Cambridge, Massachusetts, "Relation Between Conservation Laws and Path Independent Integrals in Elasticity and Thermoelasticity," presented by A. G. Herrmann, March 1982.

8. Brown University, Providence, Rhode Island, "Path-Independent Integrals for a Plane Crack, and Their Relation to General Conservation Laws in Continuum Mechanics," presented by A. G. Herrmann, March 1982